

Tensile Strengths of Glassy Plastics with Drilled Holes and Measurement of Griffith Parameters

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Synopsis

The tensile strengths of plastics with drilled holes can be analyzed by treating the hole as a nucleus in flaw generation. The analysis is accurate for hole diameters less than the inherent flaw size. Above this value the tensile strengths define a single curve when plotted on a reduced variable scale. Using the analysis in its linear range facilitates evaluation of the Griffith parameters. From such an application it was found that plasticizers markedly reduce the values of the Griffith parameters while affecting tensile strengths only slightly. Temperature has a similar effect.

Introduction

Classical elasticity theory predicts that the introduction of a circular hole into a plane bar will cause a stress concentration. If the bar is infinite the concentration factor is 3 for uniaxial tension. As the hole becomes an appreciable fraction of the bar, the concentration factor is reduced to a limiting value of 2.¹ These values are based on the nominal stress across the minimum cross section of the bar. Thus, if one were to drill holes of varying sizes into finite test bars it would be expected that the tensile strength of the material would show a sudden drop and gradual partial recovery due to the change in stress concentration factor. This is definitely not the case. Rather, the tensile strength drops and recovers in a continuous manner from the homogeneous material through the range of hole sizes.

From the above it would appear that the primary role of a small drilled hole is not that of a classical stress raiser, yet it does affect the apparent tensile strength somehow. This paper will be concerned with a theory of the effect plus an explanation of the consequences of the theory.

Theory

Let us consider two of the more familiar theories of tensile strength, the Griffith theory² and the Irwin theory.³ Both theories agree that instability occurs when the rate of strain energy release in the sample exceeds the rate of absorption of energy by the growing crack. They further agree on the form taken by the equation defining the point of instability. Where

they disagree is in the definition of the energy-absorbing term and the extent of influence of the strain-energy term.

In either of these theories an arbitrary nucleus size can be included with identical results. In the Griffith theory, the nucleus enters as a modifying term in the strain energy contribution by increasing the size of the flaw. In the Irwin theory, the nucleus adds to the inherent flaw size in the stress-intensity factor. In both cases the final equation takes the form

$$T = [4E\gamma/\pi(1 - \nu^2)(d + d_0)]^{1/2} \quad (1)$$

where T is tensile strength of the drilled sample, E is Young's modulus, γ is specific surface energy, ν is Poisson's ratio, d is the nucleus diameter, and d_0 is total inherent flaw size. It should be emphasized that γ is not restricted to the energy absorbed in bond breakage; plastic deformation energy should also be included in its definition.

If we can assume that the inherent flaw size d_0 is independent of the nucleus size then the application of the above equation is immediate. There does not appear to be any *a priori* criterion by which this assumption can be tested; therefore its justification must be based on agreement with experiment.

Recasting eq. (1) into a more usable form we obtain;

$$d + d_0 = [4E\gamma/\pi(1 - \nu^2)](1/T^2) \quad (2)$$

which permits direct evaluation of $E\gamma$ and d_0 from the slope and intercept of the d versus $1/T^2$ plot. If the relationship is of a general nature, then it should be possible to represent a large number of materials on a reduced variable basis. This relation is obtained by dividing eq. (2) for a sample with a hole (nucleus) by eq. (2) without a hole, viz.,

$$(T_0^2/T^2) - 1 = d/d_0 \quad (3)$$

T_0 being the tensile strength of the undrilled sample.

Experimental

Samples of mineral oil-modified polystyrene were prepared by blending of the two components on hot compounding rolls. The blended materials were then ground and devolatilized prior to molding. Samples were individually molded in the standard $1/2 \times 1/8$ in. tensile bar shape. A subsequent annealing step removed residual strains from the samples as evidenced by examination between crossed polaroids. The Plexiglas acrylic samples (Rohm and Haas) were machined from a $1/8$ -in. sheet of the material.

The various sizes of holes were hand-drilled to minimize thermal effects at the hole edge. The drilled samples were then tested in an Instron tensile testing machine at a strain rate of 30%/min. under ambient conditions. Other conditions of testing were used by other authors cited here;⁴⁻⁶ these conditions are listed in Table I.

TABLE I

Material	Treatment	Test rate, %/min.	T_0 , psi $\times 10^{-3}$	d_0 , in.	$E\gamma$, lb. ² /in. ³ $\times 10^5$
Plexiglas	—	30	10.1	~0.008	5.4
Styron 683	—	30	5.85	0.155	35.0
"	0.5% mineral oil	30	5.96	0.063	15.0
"	1.0% " "	30	5.79	0.045	10.0
"	1.5% " "	30	5.47	0.036	7.1
"	2.0% " "	30	5.32	0.032	6.0
Styron 666 ^a	23°C.	1.5	5.10	0.022	3.8
"	-40°C.	1.5	8.20	0.040	17.8
"	+52°C.	1.5	4.00	0.018	1.9
Styron 700 ^a	23°C.	1.5	6.70	0.014	4.2
"	-40°C.	1.5	8.20	0.030	13
Tyrl 750 ^{a,b}	—	1.5	7.20	0.017	5.8
Tyrl 767 ^{a,b}	—	1.5	7.80	0.012	4.8
Styron 666 ^c	—	1.5	4.37	0.10	13
Styron 683 ^c	—	1.5	5.34	0.14	26
Polystyrene ^d	—	—	6.5	~0.086	28
PMMA ^d	—	—	>9.5	~0.004	4

^a Data of Balazs and Cheesbro.⁴

^b Styrene-acrylonitrile copolymer (Dow Chemical Company).

^c Data of Meyer.⁵

^d Data of Berry.⁶

In all cases tensile strengths were calculated from the minimum cross-sectional area of the drilled sample.

Results

Data for a variety of glassy polymers have been plotted according to eq. (2) and are shown in Figures 1-3. Figure 1 shows the effect of a plasticizer (mineral oil) on the Griffith parameters of polystyrene. Also in Figure 1 is the same plot for poly(methyl methacrylate). This latter material shows the general shape of several curves which deviate markedly from linear behavior. It has been found empirically that linear behavior obtains up to hole sizes which equal the inherent flaw size. Poly(methyl methacrylate) has an inherent flaw size in the range of about 0.004 in.,⁷ hence it was not practical to drill holes sufficiently small to define the linear range.

The data for Figures 2 and 3 were taken from results obtained by Balazs and Cheesbro.⁴ The nonlinear regions are again well defined, but the inherent flaw sizes are large enough to allow a reasonable estimate to be made. Figure 2 shows the effect of temperature on two different polystyrenes while Figure 3 shows the behavior for several glassy materials at room temperature.

The data from all of the above have been reduced according to eq. (3) and plotted in Figure 4. The seemingly high degree of scatter in Figure 4

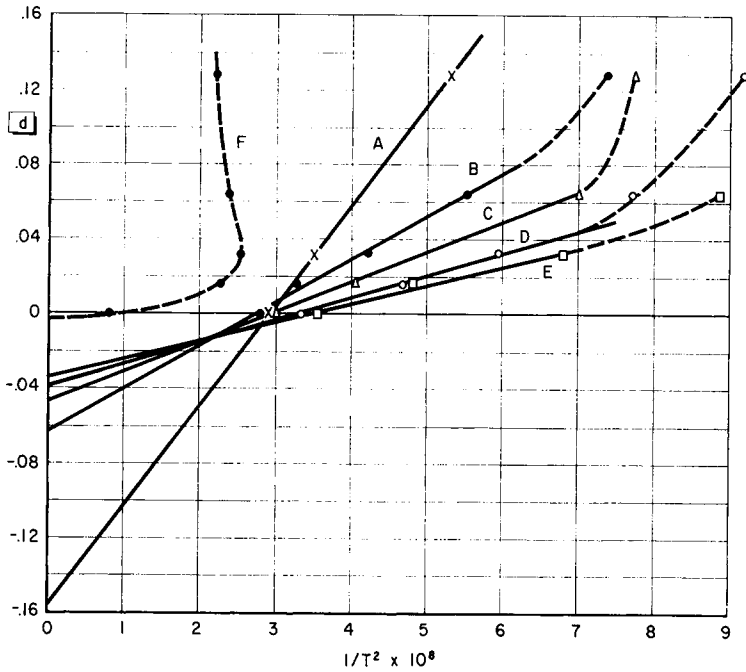


Fig. 1. Griffith plots for polystyrene with mineral oil added and for Plexiglas acrylic polymer: (A) no additive; (B) 0.5% mineral oil; (C) 1.0% mineral oil; (D) 1.5% mineral oil; (E) 2.0% mineral oil; (F) Plexiglas.

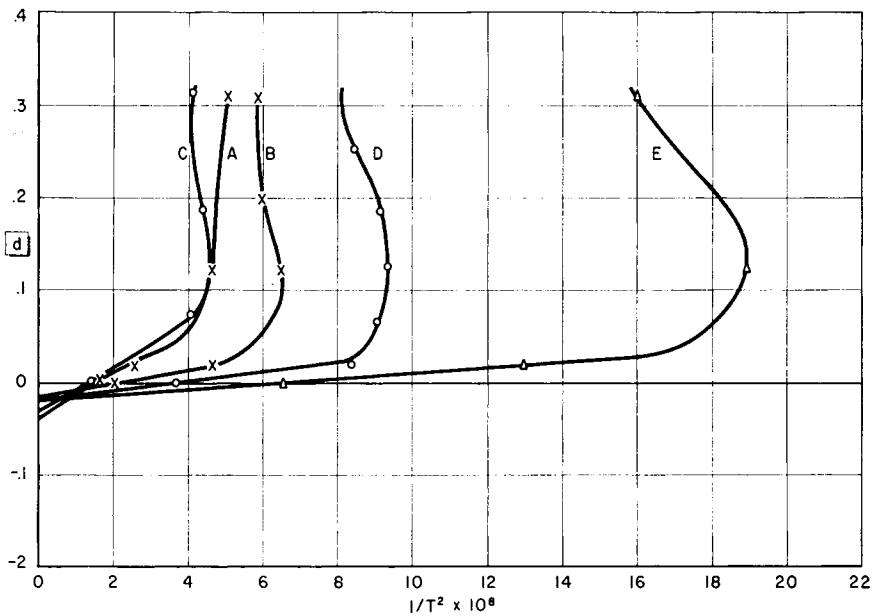


Fig. 2. Griffith plots for two Styron polystyrene products at several temperatures: (A) Styron 700 at -40°C .; (B) Styron 700 at 23°C .; (C) Styron 666 at -40°C .; (D) Styron 666 at 23°C .; (E) Styron 666 at 52°C . Data of Balazs and Cheesbro.⁴

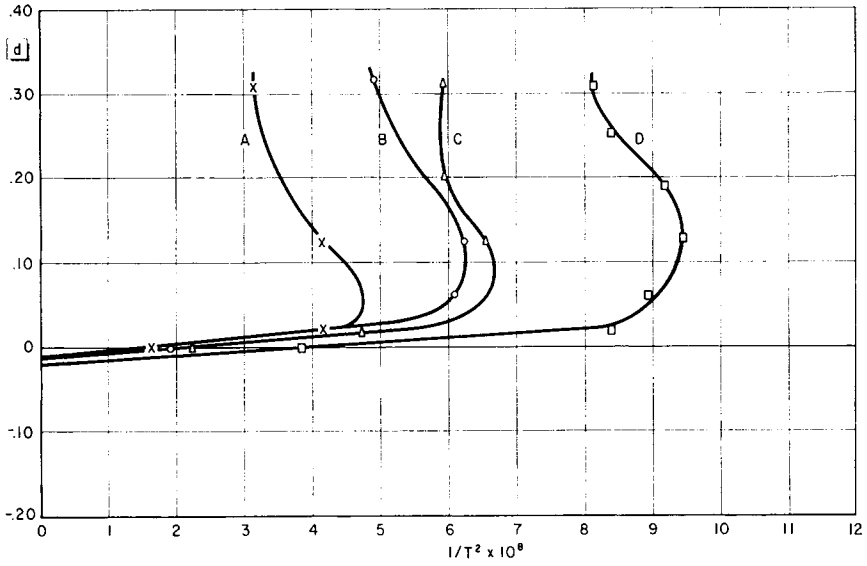


Fig. 3. Griffith plots for several glassy polymers at +23°C.: (A) Tyril 767; (B) Tyril 750; (C) Styron 700; (D) Styron 666.

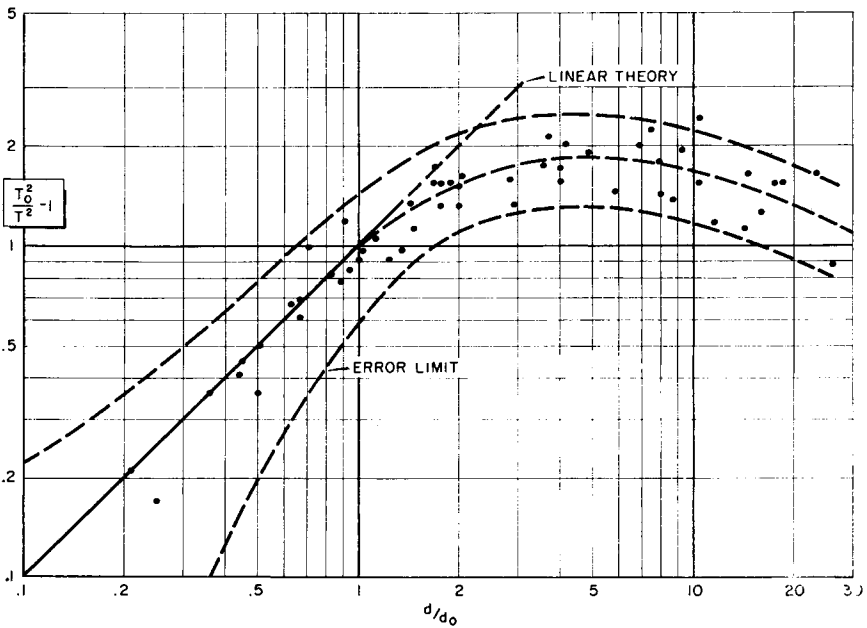


Fig. 4. Reduced variable plot for all materials from this work plus those from the literature.^{4,5}

actually falls within reasonable limits. If we assume an error in tensile strength measurements of 5%, we obtain the error limit curves shown in Figure 4. Furthermore the stress concentration pattern may well vary from material to material. For example, in poly(methyl methacrylate) a hole-to-flaw size ratio of sixteen was obtained when the hole constituted only one-fourth the sample width, whereas in Styron 666 polystyrene (Dow Chemical Company) at +52°C. a hole which was over 60% of the sample width was needed to achieve the same ratio. Thus, the general agreement between the data points is about as good as could be expected.

The numerical values of d_0 and $E\gamma$ are listed in Table I. Comparison of the values obtained by this technique with those reported by Berry⁶ shows modest agreement. In particular, the data for Styron 683 (unmodified) agree within a factor of two. Other independent measurements of the surface energy γ only agree within a factor of four.⁶ In light of the extreme sensitivity of these materials to small amounts of impurities, the agreement is satisfactory.

Discussion

From the curves in Figures 1-3 and the values in Table I, it is apparent that the Griffith parameters are extremely sensitive quantities. The addition of small amounts of plasticizer has a profound effect on the values of d_0 and $E\gamma$. Although not quite so dramatic, temperature also effects the parameter values. Furthermore, the values do not appear to change independently; rather, there seems to be a remarkable interdependence of one on the other. This opens the possibility that still other factors may be fundamental and the Griffith parameters merely derivatives.

It is known that, for polymers, most of the surface energy term γ must be identified with some mechanism other than bond breakage.⁸ This identification is usually made with plastic flow. Plastic flow in turn can be invoked in describing craze formation.⁹ Furthermore, it has been observed that the length of a craze plane just prior to failure correlates well with the extrapolated values of d_0 .¹⁰ On the basis of these observations, it is reasonable to look to craze formation as a precursor to the ultimate flaw. Thus, those factors which would be expected to influence the energy absorbed in craze formation probably define the ultimate flaw size as well. The most difficult point to explain is the apparent insensitivity of d_0 to the inclusion of small drilled holes. This fact suggests that some mechanism is operative which defines a quasi-equilibrium craze size based only on the actual craze area and independent of a stress concentrator near the center of the craze. It is not possible at this moment to define the mechanism, but it seems probable that such a mechanism would include an energy coupling between adjacent chains near the bulk-craze interface. Such a coupling could be invoked to explain the effects of temperature and plasticizer content on the Griffith parameters.

Conclusions

- (1) The Griffith parameters for polystyrene are extremely sensitive to plasticizer content and moderately sensitive to temperature.
- (2) The values of these parameters are probably not independent, but may derive from other, more fundamental, parameters.
- (3) In glassy polymers, craze planes precede and define the extent of the Griffith flaw.
- (4) An empirical, reduced variable plot can be used to predict the behavior of glassy plastics with holes drilled in them.

References

1. E. V. Condon and H. Odishaw, *Handbook of Physics*, McGraw-Hill, New York, 1958, p. 3-88.
2. A. A. Griffith, *Phil. Trans. Roy. Soc.*, **A221**, 163 (1921).
3. G. R. Irwin, *J. Appl. Mech.*, **79**, 361 (Sept. 1957).
4. C. F. Balazs and A. M. Cheesbro, Dow Publication P. D. and S. No. 6364-4.
5. V. E. Meyer, private communication.
6. J. P. Berry, *J. Polymer Sci.*, **50**, 313 (1961).
7. J. P. Berry, *J. Polymer Sci.*, **50**, 107 (1961).
8. J. P. Berry, in *Fracture Processes in Polymeric Solids*, B. Rosen, Ed., Interscience, New York, 1964, p. 196.
9. O. K. Spurr, Jr., and W. D. Niegisch, *J. Appl. Polymer Sci.*, **6**, 585 (1962).
10. C. B. Arends, unpublished observations.

Résumé

Les résistances à la traction de plastiques perforés peuvent être analysées en considérant le trou comme un noyau de départ pour la formation de la fissure. L'analyse est précise pour un trou de diamètre inférieure à la grandeur de la déchirure inhérente. Pour des valeurs supérieures à celle-ci des résistances à la traction définissent une courbe unique quand les valeurs sont reportées sur un diagramme à échelle variable réduite. Quand on emploie l'analyse dans sa partie linéaire, on facilite l'évaluation des paramètres de Griffith. Au départ d'une telle application on a trouvé que les plastifiants diminuent considérablement les valeurs des paramètres de Griffith alors qu'il n'affectent les résistances à la traction que faiblement. La température exerce un effet similaire.

Zusammenfassung

Eine Analyse der Zugfestigkeit von Plastomeren mit gebohrten Löchern kann durch Behandlung des Lochs als Keim zur Fehlstellenbildung durchgeführt werden. Die Analyse ist für Lochdurchmesser kleiner als die spezifische Fehlstellengröße streng gültig. Oberhalb dieses Wertes erhält man bei Auftragung der Zugfestigkeitswerte in einem Diagramm mit reduzierten Variablen eine einzige Kurve. Die Verwendung der Analyse in ihrem linearen Bereich erleichtert die Ermittlung der Griffith-Parameter. Bei dieser Anwendung zeigte sich, dass Weichmacher den Wert der Griffith-Parameter merklich herabsetzen, während sie einen nur geringen Einfluss auf die Zugfestigkeit haben. Die Temperatur hat einen ähnlichen Einfluss.

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